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On the (im)possibility of warp bubbles

Chris Van Den Broeck*
Physics Division, Starlab Research
Boulevard St.–Michel 47, 1040 Brussels, Belgium

Abstract

Various objections against Alcubierre's warp drive geometry are reviewed. Superluminal warp bubbles seem an unlikely possibility within the framework of general relativity and quantum field theory, although subluminal bubbles may still be possible.

* vdbroeck@starlab.net

1 Introduction

Since Alcubierre published his 'warp drive' spacetime [1], the proposal has been criticized from various viewpoints by a number of authors [2, 3, 4, 5]. One of the problems, concerning the amount of exotic matter needed to support a warp bubble capable of transporting macroscopic objects [3], was partially solved in [6]. Another objection, claiming a divergence of quantum fluctuations on a warp drive background, is probably not valid in the general case [4]. However, serious problems remain, and as we will see, it is unlikely that the original ansatz can be modified to circumvent all of them, at least for superluminal bubbles.

In the next section, the warp drive geometry is introduced; in the subsequent section, we review some of the objections that have been raised. Section 3 deals with the behaviour of quantum fluctuations on a fixed Alcubierre background. In section 4 we discuss the unreasonably high energies macroscopic warp bubbles would need. In section 5 we come to the crucial problem: part of the energy supporting the warp drive moves tachyonically. A summary is given in section 6.

2 The Alcubierre spacetime

The warp drive metric is

$$ds^2 = -dt^2 + (dx - v_s(t) f(r_s) dt)^2 + dy^2 + dz^2, \tag{1}$$

with $r_s = \sqrt{(x - x_s(t))^2 + y^2 + z^2}$, and $v_s = \frac{dx_s}{dt}$, where $x_s(t)$ is the path followed by the center of the warp bubble. The function f has the properties $f(0) = 1$ and $f(r_s) \to 0$ as $r_s \to \infty$. $x_s(t)$ is then a timelike geodesic with proper time equal to the coordinate time outside the bubble. We will assume that f has compact support, which we will call the warp bubble. This is a natural assumption since it implies that the energy densities associated to the geometry do not stretch all the way to spacelike infinity.

At first sight there is no speed limit, in the sense that if $v_s > 1$, a particle moving along $x_s(t)$ would be able to outrun a photon moving in the Minkowskian part of spacetime. This is also characteristic of traversable wormholes [7], but unlike wormholes, the warp drive does not need non–trivial topology. However, it will become clear that as soon as the bubble goes superluminal, the geometry (1) develops unphysical features, not all of which can be mended by simple modifications of the spacetime.

A problem the warp drive has in common with traversable wormholes is a violation of the energy conditions of general relativity. If quantum field theory (QFT) is introduced, this is no longer a crucial problem; for example, the well–known Casimir effect violates the Weak Energy Condition (WEC)[1]. As has been known for a long time, QFT on curved spacetimes indicates that spacetime curvature itself could cause violations of the energy conditions, and recently a class of wormholes was found which would self–stabilize, in the sense that the negative energy ('exotic matter') densities needed to sustain the wormhole geometry would arise from vacuum fluctuations of conformal fields due to the curvature of the wormhole geometry itself [9].

3 Quantum fields on an Alcubierre background

Hiscock [4] argued that the energy density due to fluctuations of conformally coupled quantum fields would diverge at particle horizons within the bubble, which are present as soon as $v_s > 1$. The calculation was only performed for the $1+1$ dimensional version of the warp drive geometry, but it is reasonable to assume that a similar phenomenon would occur in four dimensions [10]. Hiscock's calculations involved a coordinate transformation making the warp drive spacetime manifestly static. In two dimensions, the geometry turned out to be similar to that of a 2–dimensional black hole. Calculations of the stress–energy tensor of a conformal field living on such a background [11] indicate that if the field has reached thermal equilibrium, its temperature at spacelike infinity must be equal to the Hawking temperature of the black hole, otherwise vacuum fluctuations would diverge strongly at the horizon. In the case of the constant velocity warp drive, the temperature of the horizon would never be equal to that of a field on the background, from which a divergence was inferred. However, it is questionable that such a divergence would be present if the warp bubble had gone superluminal and developed horizons a finite time in the past, a situation comparable to that of a newly formed black hole not in thermal equilibrium with the field at infinity.

4 Unreasonably high energies

Ford and Roman [12] suggested an uncertainty–type principle which places a bound on the extent to which the WEC is violated by quantum fluctuations of scalar and electromagnetic fields: The larger the violation, the shorter the time it can last for an inertial observer crossing the negative energy region. This so–called quantum

[1]Helfer et al. [8] argued that this would not be true in realizable experimental set–ups due to the properties of known materials, but their results do not in principle rule out a Casimir WEC violation.

inequality (QI) can be used as a test for the viability of would–be spacetimes allowing superluminal travel. By making use of the QI, Ford and Pfenning [3] were able to show that a warp drive with a macroscopically large bubble must contain an unphysically large amount of negative energy. This is because the QI restricts the bubble wall to be very thin, and for a macroscopic bubble the energy is roughly proportional to R^2/Δ, where R is a measure for the bubble radius and Δ for its wall thickness. It was shown that a bubble with a radius of 100 meters would require a total negative energy of at least

$$E \simeq -6.2 \times 10^{62} v_s \text{ kg}, \tag{2}$$

which, for $v_s \simeq 1$, is ten orders of magnitude bigger than the total positive mass of the entire visible Universe.

In [6], it was shown that this number is very much dependent on the details of the geometry. The total energy can be reduced dramatically by keeping the surface area of the warp bubble itself microscopically small, while at the same time expanding the spatial volume inside the bubble. The most natural way to do this is the following:

$$ds^2 = -dt^2 + B^2(r_s)[(dx - v_s(t) f(r_s) dt)^2 + dy^2 + dz^2]. \tag{3}$$

$B(r_s)$ is a twice differentiable function such that, for some $\tilde{R}$ and $\tilde{\Delta}$,

$$\begin{aligned} B(r_s) = 1 + \alpha \quad & \text{for} \quad r_s < \tilde{R}, \\ 1 < B(r_s) \leq 1 + \alpha \quad & \text{for} \quad \tilde{R} \leq r_s < \tilde{R} + \tilde{\Delta}, \\ B(r_s) = 1 \quad & \text{for} \quad \tilde{R} + \tilde{\Delta} \leq r_s, \end{aligned} \tag{4}$$

where α will in general be a very large constant; $1 + \alpha$ is the factor by which space is expanded. For f one chooses a function with the properties

$$\begin{aligned} f(r_s) = 1 \quad & \text{for} \quad r_s < R, \\ 0 < f(r_s) \leq 1 \quad & \text{for} \quad R \leq r_s < R + \Delta, \\ f(r_s) = 0 \quad & \text{for} \quad R + \Delta \leq r_s, \end{aligned}$$

where $R > \tilde{R} + \tilde{\Delta}$.

A spatial slice of the geometry one gets in this way can be easily visualized in the 'rubber membrane' picture. A small Alcubierre bubble surrounds a neck leading to a 'pocket' with a large internal volume, with a flat region in the middle. It is easily calculated that the center $r_s = 0$ of the pocket will move on a timelike geodesic with proper time t.

Using this scheme, the required total energy can be reduced to stellar magnitude, in such a way that the QI is satisfied. On the other hand, the energy densities are still unreasonably large, and the spacetime has structure with sizes only a few orders of magnitude above the Planck scale.

5 Energy moving locally faster than light

The most problematic feature of the warp drive geometry is the behaviour of the negative energy densities in the warp bubble wall [2, 5]. If the Alcubierre spacetime is taken literally, part of the exotic matter will have to move superluminally *with respect to the local lightcone*. It is easy to see that all exotic matter outside some surface surrounding the center (let us call this the critical surface), will move in a spacelike direction. For $v_s > 1$, there has to be exotic matter outside the critical surface, since the function f must reach the value 0 for some r_s (which, of course, can be infinity), and the negative energy density is proportional to $\left(\frac{df}{dr_s}\right)^2$ for an 'Eulerian observer' [1]. As noted in [5], the Alcubierre spacetime is an example of what can happen when the Einstein equations are run in the 'wrong' direction, first specifying a metric, then calculating the associated energy–momentum tensor.

The problem of tachyonic motion can be interpreted as meaning that part of the necessary exotic matter is not able to keep up with the rest of the bubble: if one would try to make a warp bubble go superluminal, the outer shell would be left behind, destroying the warp effect.

It is conceivable that the problem can be circumvented, for example by letting the distribution of exotic matter expand into a 'tail' in the back. It may be possible to do this in a way compatible with both the QI and the Quantum Interest Conjecture introduced in [13], which states that a pulse of negative energy must always be followed by a larger pulse of positive energy. However, it is unlikely that one can get rid of tachyonic motion of the exotic matter without introducing a naked curvature singularity in the front of the bubble [14].

6 Summary

It would seem that the main problems of the warp drive can not be solved without retaining some unphysical features or introducing new ones, such as high energy densities, curvature radii of the order of the Planck length, and naked singularities. We have limited the discussion to superluminal warp bubbles. Subluminal bubbles are still an open possibility, and it is not inconceivable that microscopic ones might even occur naturally. Due to the absence of horizons, potential problems due to diverging vacuum fluctuations will not arise, and there will be no tachyonic motion of exotic matter. Possibly the geometry can be chosen in such a way that the necessary negative energy densities are partly supplied by the changes in vacuum fluctuations induced by the curvature, as in [9]. An interesting question is whether one can construct a spacetime similar to the subluminal warp drive which avoids negative energy densities altogether, but for this no ansatz is available at the present time.

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